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Estimation of Product Moment Correlation Coefficients Through the Use of the Ratio of Contingency Coefficient to the Maximal Contingency Coefficient[1]

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Over sixty years ago, Pearson (1904) in his fundamental paper on the theory of contingency clearly indicated some of the difficulties of comparing the coefficients of relationship, correlation and contingency. Pearson did show that, with certain reservations concerning fineness of subdivision in classification, the coefficient of contingency is essentially identical with the product moment correlation coefficient as deduced from a normal correlation surface.

In a practical sense, contingency coefficients are not directly comparable unless derived from the same size contingency tables and they are not directly comparable to product moment correlation coefficients because of a limitation regarding upper limits and because of a measurement restriction problem. The upper limits for contingency coefficients are a function of the number of categories. The upper limit for a 2 X 2 table is .707; for a 3 X 3 table, .816; for a 4 X 4 table, .866; for a k X k table, $\sqrt{(k - 1)/k}$.

Over four decades ago, Kelley (1924) presented corrections which may be applied to make contingency coefficients estimates of product moment correlations.[2] The corrections are most tedious and time consuming to make. One correction is for number of categories. The other correction requires the assumptions that the underlying traits are continuous and normal in distribution. McNemar (1962, p. 201) suggests that if the assumptions of normally distributed continuous variables are tenable and if one is justified in reducing a more than four-cell contingency table to a 2 X 2 table, one can instead determine the value of tetrachoric r.

The purpose of the present paper is to suggest another and more simplified approach to use when one desires to compare a contingency coefficient to a product moment correlation coefficient. This approach is not dissimilar

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[1]Based on a paper read at the Psychometric Society meeting, September 2, 1966, New York.

[2]The need for correcting contingency coefficients has also been shown by Harris and Treloar (1927) and by Harris and Chi Tu (1929).

to what some investigators, namely those using coefficients of correlation in factor analysis, have done to make the phi coefficient supposedly comparable to r by computing the ratio, ϕ/ϕ_{max}, when the ϕ_{max} value has been determined by an equation developed by either Ferguson (1941) or Guilford (1965) involving the marginal means, p_i and p_j

We will attempt to empirically demonstrate that the ratio, contingency coefficient/maximal contingency coefficient (C/C_{max}), is also directly comparable to the product moment correlation coefficient. The bivariate data used in this investigation were obtained from 45 statistics textbooks. Product moment correlation coefficients were computed using 74 sets of bivariate data (Ns ranged from 20 to 6835 and rs ranged from 0.00 to 1.00). When data were cast into 2 X 2 contingency tables, an attempt was always made so as to have dichotomies as near to .50--.50 proportions as possible. However, the achievement of such .50--.50 proportions was seldom possible. The dichotomization was also done so that no contingency table cell would have an expected value of less than five.

Inequality of means in correlated, dichotomized variables has an effect upon the size of a contingency coefficient computed from such bivariate data. The data from 38 of the 74 bivariate data sets were recast into 2 X 2 tables so that the marginal proportions, p_i, q_i, p_j, and q_j would vary widely. However, adherence to the restriction that expected values for cells must not be less than five was followed. Some of these bivariate data sets were cast into as many as 11 different 2 X 2 contingency tables. With each data set, the C/C_{max} ratio which best approximated the computed correlation coefficient was selected. These selected C/C_{max} ratios were then statistically compared to the product moment correlation coefficients. The product moment correlation coefficient between the selected C/C_{max} ratio values and the correlation coefficients was high (r = .934, N = 74, p <.001). As should be expected there was a very high linear relationship between values from these two relationship indices. The intraclass correlation coefficient between these two sets of relationship estimation values was only slightly lower (R = .924, p <.001) and represented an estimate of agreement between the two sets of relationship estimations when they had been classified in 20 groups in which the interval size was .05, *e.g.*, .00 - .04, .05 - .09, .10 - .14, etc.

The test for the difference between the product moment correlation coefficient (mean r = .592, *S.D.* = .257) and the C/C_{max} ratio (mean C/C_{max} = .558, *S. D.* = .247) was significant (*C.R.*= 3.97, p <.001). It appeared that the C/C_{max} ratio model provided a conservative estimate of the correlation coefficient.

Inspection of all of the computed C/C_{max} ratios, from 2 X 2 tables, (see Table 1) showed that the ratios which corresponded most closely to the product moment correlation coefficients were not always the ones which were associated with fourfold tables having dichotomies nearer to .50 - .50 proportions. However, we're lead to believe that the C/C_{max} ratios which best approximated the product moment correlation coefficients generally were from the fourfold tables where $p_i \cong p_j \cong .50$.

Twenty of the 74 bivariate data sets were also cast into 3 X 3 tables. Contingency coefficients and C/C_{max} ratios were computed and were compared to the product moment correlation coefficients. With 10 of these bivariate data sets, the C/C_{max} ratios when compared to the product moment correlation coefficients were less adequate than when the C/C_{max} ratios were computed from 2 X 2 tables. Two of the bivariate data sets were also cast into 4 X 4 tables, contingency coefficients and C/C_{max} ratios were computed, and were compared to the product moment correlation coefficients. One of these two C/C_{max} ratios represented a more accurate estimate of the correlation coefficient than did the C/C_{max} ratios computed from 2 X 2 and 3 X 3 tables. From this limited evidence it cannot be said that the C/C_{max} ratio computed from 3 X 3 or 4 X 4 tables provide more accurate estimates of the product moment correlation coefficients than those C/C_{max} ratios computed from 2 X 2 tables.

The implications of these conclusions for the use of the C/C_{max} ratio are not clear. However, it would appear, based on this empirical demonstration, that the C/C_{max} ratio may be used as a "quick and dirty" estimate of the relationship measure provided by the product moment correlation model. No mathematical justification is offered for this contingency coefficient ratio, C/C_{max}. However, it has been pointed out by Guilford (1965) that he has not seen any mathematical justification regarding the ratio, ϕ/ϕ_{max}, as an index of relationship and it has received wide use as a statistical device.

Table 1

Relationship Statistics, r and C/C_{max}, Computed with

Differing Marginal Values (Arranged According to N Size)

N	r	C/C_{max}	p_i	q_i	p_j	q_j	N	r	C/C_{max}	p_i	q_i	p_j	q_j
20	.60	.69	.40	.60	.40	.60	56	.72	.71	.59	.41	.52	.48
		.51	.40	.60	.65	.35			.63	.36	.64	.38	.62
32	.53	.48	.69	.31	.56	.44			.53	.29	.71	.23	.77
		.44	.47	.53	.56	.44	64	.00	.02	.69	.31	.69	.31
		.43	.47	.53	.50	.50			.05	.69	.31	.94	.06
		.69	.69	.31	.69	.31	64	.25	.24	.94	.06	.31	.69
		.76	.78	.22	.78	.22			.23	.69	.31	.31	.69
		.20	.12	.88	.12	.88			.18	.69	.31	.69	.31
35	.43	.36	.37	.63	.37	.63			.10	.94	.06	.69	.31
		.35	.49	.51	.51	.49	64	.50	.46	.69	.31	.69	.31
		.52	.60	.40	.66	.34			.42	.31	.69	.69	.31
		.30	.37	.63	.40	.60			.33	.94	.06	.69	.31
40	.68	.56	.42	.58	.30	.70			.28	.94	.06	.94	.06
		.54	.68	.32	.72	.28			.24	.06	.94	.69	.31
49	.97	.93	.83	.17	.80	.20			.09	.06	.94	.94	.06
		.88	.55	.45	.61	.39	64	.75	.69	.69	.31	.69	.31
		.84	.41	.59	.41	.59			.60	.94	.06	.94	.06
		.78	.41	.59	.61	.39			.59	.31	.69	.69	.31
		.44	.69	.31	.94	.06			.43	.34	.66	.69	.31
		.17	.06	.94	.69	.31			.41	.34	.66	.48	.52
64	1.00	1.00	.69	.31	.69	.31	92	.67	.64	.50	.50	.55	.45
		1.00	.94	.06	.94	.06			.63	.28	.72	.29	.71
65	.76	.53	.37	.63	.35	.65			.79	.38	.62	.38	.62
		.47	.37	.63	.65	.35	99	.24	.12	.54	.46	.56	.44
69	.93	.89	.36	.64	.36	.64			.04	.34	.66	.29	.71
		.88	.49	.51	.54	.46			.03	.19	.81	.14	.86
		.78	.23	.77	.20	.80			.46	.54	.46	.43	.57
		.76	.09	.91	.09	.91	100	.38	.38	.50	.50	.54	.54

Table 1

(continued)

N	r	C/C_{max}	p_i	q_i	p_j	q_j	N	r	C/C_{max}	p_i	q_i	p_j	q_j
72	.75	.75	.44	.56	.53	.47			.33	.17	.83	.12	.88
		.76	.44	.56	.46	.54			.22	.33	.67	.32	.68
75	.07	.08	.40	.60	.67	.33	100	.82	.83	.50	.50	.56	.44
		.11	.61	.39	.67	.33			.80	.81	.19	.76	.24
		.03	.61	.39	.33	.67			.75	.72	.28	.63	.37
		.00	.40	.60	.33	.67			.71	.50	.50	.45	.55
85	.54	.52	.69	.31	.69	.31	100	1.00	1.00	.46	.54	.46	.54
		.51	.69	.31	.48	.52			1.00	.64	.36	.64	.36
		.60	.92	.08	.85	.15			1.00	.79	.21	.79	.21
106	.78	.79	.27	.73	.27	.73	140	.59	.61	.16	.84	.62	.38
		.79	.40	.60	.27	.73			.56	.33	.67	.46	.54
		.85	.40	.60	.44	.56			.69	.33	.67	.62	.38
		.69	.27	.73	.44	.56			.44	.16	.84	.46	.54
		.67	.65	.35	.44	.56	141	.82	.76	.59	.41	.50	.50
		.56	.93	.07	.91	.09			.73	.75	.25	.60	.40
		.56	.65	.35	.58	.42			.69	.39	.61	.42	.58
		.54	.40	.60	.58	.42			.65	.39	.61	.50	.50
110	.13	.09	.62	.38	.56	.44	149	.68	.65	.42	.58	.51	.49
		.20	.45	.55	.56	.44			.58	.58	.42	.42	.58
		.06	.26	.74	.32	.68			.57	.71	.29	.29	.71
		.03	.36	.64	.41	.59			.49	.81	.19	.20	.80
113	.37	.36	.73	.27	.71	.29	188	.03	.06	.68	.32	.61	.39
		.44	.62	.38	.60	.40			.07	.90	.10	.71	.29
		.46	.50	.50	.45	.55			.10	.44	.56	.52	.48
120	.60	.52	.65	.35	.66	.34	192	.08	.07	.43	.57	.64	.36
		.50	.38	.62	.66	.34			.04	.43	.57	.36	.64
		.49	.38	.62	.35	.65	192	.48	.45	.39	.61	.58	.42
		.47	.65	.35	.35	.65			.54	.48	.52	.41	.59
193	.79	.86	.51	.49	.38	.62	281	.55	.56	.31	.69	.30	.70
		.70	.51	.49	.76	.24			.57	.41	.59	.40	.60

Table 1

(continued)

N	r	C/C_{max}	p_i	q_i	p_j	q_j
		.70	.23	.77	.38	.62
		.89	.51	.49	.61	.39
		.90	.69	.31	.61	.39
		.91	.69	.31	.76	.24
		.60	.23	.77	.19	.81
		.60	.51	.49	.19	.81
		.57	.23	.77	.61	.39
		.46	.51	.49	.89	.11
		.40	.51	.49	.09	.91
193	.86	.82	.45	.55	.48	.52
		.82	.43	.57	.47	.53
202	.57	.57	.70	.30	.27	.73
		.58	.44	.56	.46	.54
		.62	.58	.42	.46	.54
225	.80	.77	.79	.21	.20	.80
		.76	.56	.44	.40	.60
		.85	.45	.55	.50	.50

N	r	C/C_{max}	p_i	q_i	p_j	q_j
		.52	.21	.79	.20	.80
		.52	.50	.50	.50	.50
310	.69	.59	.47	.53	.45	.55
		.56	.71	.29	.67	.33
		.53	.86	.14	.77	.23

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MULTIPLE COMPARISONS IN A REGRESSION FRAMEWORK

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In an analysis of variance framework, a great deal of effort has been expended in the past two decades with the multiple comparisons situation. Essentially, the concern has been to preserve the probability level in the experimental situation, and still make additional tests involving the means, in addition to the main effects test that is usually made in a one-way layout.

Within the analysis of variance framework, several tests for multiple comparison have been devised. Dunnett (1955, 1964) constructed a test applicable to the situation in which several experimental groups are to be compared to a control group. Duncan's (1955) test is useful to comparing each mean to every other mean. Dunn (1961) devised a test which would retain maximum power if a limited number of comparisons are of interest and are decided upon on an _a priori_ basis. A test which is useful on an _a posteriori_ basis is Scheffé's (1953) test. This test is amenable to data snooping, but has the drawback of losing power, as compared to the other methods.

Each of the previously mentioned tests require either additional tables or, in the case of Scheffé's test, a modification of the usual tables for the F test. On the other hand, these same tests can be achieved by using multiple regression as a problem solving technique. There is one logical extension here: _appropriate tables should be consulted_. This point will be elaborated on in more detail later.

An Illustrative Example

Suppose the following information were available on four random groups:

GROUP I	GROUP II	GROUP III	GROUP IV
9	8	13	15
8	7	10	12
6	8	12	10
3	6	11	17
4	6	14	11
$\bar{X}_1 = 6.0$	$\bar{X}_2 = 7.0$	$\bar{X}_3 = 12.0$	$\bar{X}_4 = 13.0$

The different types of multiple comparison procedures involve different hypotheses (i.e. restrictions). The various types of multiple comparison procedures to be considered in this paper are the following: Duncan's multiple range test, Dunn's "c" test, and Scheffé's test. Dunnett's test for several comparisons with a control has been treated elsewhere (Williams, in press).

Duncan's Multiple Range Test

For the data presented in the illustrative example, there are $\binom{4}{2}$, or 6, comparisons of interest (that is, all possible contrasts of pairs) for Duncan's multiple range test. They are the following:

$\bar{X}_1$ to $\bar{X}_2$

$\bar{X}_1$ to $\bar{X}_3$

$\bar{X}_1$ to $\bar{X}_4$

$\bar{X}_2$ to $\bar{X}_3$

$\bar{X}_2$ to $\bar{X}_4$

$\bar{X}_3$ to $\bar{X}_4$

The full model for the data in the example is:

$$Y = b_0U + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + E \quad (1)$$

where

U = a unit vector

X_1 = 1 if the score is from a member of Group 1; and 0 otherwise

X_2 = 1 if the score is from a member of Group II; and 0 otherwise

X_3 = 1 if the score is from a member of Group III; and 0 otherwise

X_4 = 1 if the score is from a member of Group IV; and 0 otherwise

b_0 - b_4 are the regression coefficients determined by the least squares method

E_1 = the error involved in prediction

Restricted models in the regression framework are easily developed. For example, for the hypothesis $\overline{X}_1 = \overline{X}_2$, if the regression coefficients are equated in the full model ($b_1 = b_2 = b_6$), then the restricted model can be found:

$$Y = b_5U + b_6X_1 + b_6X_2 + b_6X_3 + b_6X_4 + E_2$$

$$Y = b_5U + b_6(X_1 + X_2) + b_7X_3 + b_8X_4 + E_2 \qquad (2)$$

Let V_1 =1 if the score is from a member of <u>either</u> X_1 or X_2; and 0 otherwise

Then equation (2) can be transformed:

$$Y = b_5U + b_6V_1 + b_7X_3 + b_8X_4 + E_2 \qquad (3)$$

Equation (3) in the <u>restricted model</u> for the hypothesis $\overline{X}_1 = \overline{X}_2$. Similar restricted models can be written for the remaining five comparisons. To make this more specific, Table 1 contains a useful formulation for this situation.

Table 1

A Regression Formulation of Duncan's Multiple Range Test

Y	U	X_1	X_2	X_3	X_4	V_1	V_2	V_3	V_4	V_5	V_6
9	1	1	0	0	0	1	1	1	0	0	0
8	1	1	0	0	0	1	1	1	0	0	0
6	1	1	0	0	0	1	1	1	0	0	0
3	1	1	0	0	0	1	1	1	0	0	0
4	1	1	0	0	0	1	1	1	0	0	0
8	1	0	1	0	0	1	0	0	1	1	0
7	1	0	1	0	0	1	0	0	1	1	0
8	1	0	1	0	0	1	0	0	1	1	0
6	1	0	1	0	0	1	0	0	1	1	0
6	1	0	1	0	0	1	0	0	1	1	0
13	1	0	0	1	0	0	1	0	1	0	1
10	1	0	0	1	0	0	1	0	1	0	1
12	1	0	0	1	0	0	1	0	1	0	1
11	1	0	0	1	0	0	1	0	1	0	1
14	1	0	0	1	0	0	1	0	1	0	1
15	1	0	0	0	1	0	0	1	0	1	1
12	1	0	0	0	1	0	0	1	0	1	1
10	1	0	0	0	1	0	0	1	0	1	1
17	1	0	0	0	1	0	0	1	0	1	1
11	1	0	0	0	1	0	0	1	0	1	1

To make the comparison of $\overline{X}_1$ to $\overline{X}_2$, the following equation can be used:

$$F' = \frac{\left(R^2_{FM} - R^2_{RM}\right)/1}{\left(1 - R^2_{FM}\right)/ df_w} \qquad (4)$$

The R^2_{FM} is a term used for the square of the multiple correlation coefficient in the <u>full model</u>, and R^2_{RM} is a term for the square of the multiple correlation coefficient in the <u>restricted model</u>. The df_w term is equivalent to the degrees of freedom for within in an analysis of variance. situation; in the present situation, $df_w = 16$.

For the present comparison,

$R_{FM} = .84516$, and $R^2_{FM} = .71429$ ·

$R^2_{RM} = .83942$, and $R^2_{RM} = .70463$ ·

Using equation (4), $F' = .5414$

The focal question centers upon the evaluation of this number. One approach is simply to compare it to the F distribution with 1 and 16 degrees of freedom. Because the F distribution with 1 and k degrees of freedom is equal to t^2, it can be seen that, by using the F distribution in a straightforward manner, the evaluation has the same inherent problems as the usual t test. Also, in using Duncan's test, the experimenter <u>knows</u> he is going to make $\binom{n}{2}$ comparisons. Before answering directly the question concerning the evaluation of the outcome of $F' = .5414$, the other comparisons of interest are made.

A second comparison of interest in using Duncan's test is comparing $\overline{X}_1$ to $\overline{X}_3$:

The full model is:

$Y = b_0U + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + E_1$ (1)

With the restriction $b_1 = b_3 = b_{10}$,

$Y = b_9U + b_{10}X_1 + b_{11}X_2 + b_{10}X_3 + b_{12}X_4 + E_3$

$Y = b_9U + b_{10}(X_1 + X_3) + b_{11}X_2 + b_{12}X_4 + E_3$

Let $V_2 = 1$ if the score is from either a member of X_1 or X_3; 0 otherwise

Then

$Y = b_9U + b_{10}V_2 + b_{11}X_2 + b_{12}X_4 + E_3$ (5)

Equation (5) is the restricted model for the hypothesis $\overline{X}_1 = \overline{X}_3$; $R = .60564$ and $R^2 = .36680$. $F' = 19.4602$.

The additional comparisons were made by going through this procedure four more times.

For the comparison of $\overline{X}_1$ to $\overline{X}_4$, for the restricted model, $R = .49124$, and $R^2 = .24132$, with $F' = 26.4875$;

For the comparison of $\overline{X}_2$ to $\overline{X}_3$, for the restricted model, $R = .68773$, and $R^2 = .47297$, with $F' = 13.5144$;

For the comparison of $\overline{X}_2$ to $\overline{X}_4$, for the restricted model, $R = .60564$, and $R^2 = .36680$, with $F' = 19.4602$;

For the comparison of $\overline{X}_3$ to $\overline{X}_4$, for the restricted model, $R = .83943$, and $R^2 = .70464$, with $F' = .5414$.

Before interpreting these calculations, it is worthwhile to order the groups concerning the size of the means. The order from low to high is the same as the subscripts; that is, $\overline{X}_1$ is the lowest, $\overline{X}_2$ is the second lowest, $\overline{X}_3$ is next to highest, and $\overline{X}_4$ is highest.

To evaluate these calculations, in each case, the square root of the F' value is found. This number is then compared with the appropriate number from Duncan's tables (Duncan's tables can also be found in Edwards, 1968).

This is an important point: to make appropriate probability statements concerning the outcome of a series of comparisons, an appropriate table should be used. When making more than one comparison, the only times the F distribution could be directly used occur when the comparisons are orthogonal; even this concession to using the F distribution is sometimes disputed.

Table 2 summarizes the comparisons, using Duncan's multiple range test.

Table 2

Duncan's Multiple Range Test in a Regression Formulation

Comparison	F'	$\sqrt{F'} = t$	Region of Rejection at .05 level	Decision
$\bar{X}_1$ to $\bar{X}_2$	.5414	.735	$t \geqq 2.469$	retain H_0
$\bar{X}_1$ to $\bar{X}_3$	19.4602	4.411	$t \geqq 2.596$	reject H_0
$\bar{X}_1$ to $\bar{X}_4$	26.4875	5.147	$t \geqq 2.673$	reject H_0
$\bar{X}_2$ to $\bar{X}_3$	13.5144	3.680	$t \geqq 2.469$	reject H_0
$\bar{X}_2$ to $\bar{X}_4$	19.4602	4.411	$t \geqq 2.596$	reject H_0
$\bar{X}_3$ to $\bar{X}_4$	.5414	.736	$t \geqq 2.469$	retain H_0

If the F distribution had erroneously been used, the region of rejection would be $t = \sqrt{F_{1,16}} = \sqrt{4.49} = 2.12$. Thus, by using the tables for Duncan's multiple range test, it is less probable for the null hypothesis to be rejected. Of course, this is to be expected.

Dunn's "c" Test

Dunn's "c" test allows for a powerful multiple comparison method when the comparisons are planned beforehand and are few in number. Suppose the following four comparisons are of interest:

$\bar{X}_1$ to $\bar{X}_2$

$\bar{X}_3$ to $\bar{X}_4$

$\bar{X}_1$ to $\bar{X}_3$

$\bar{X}_1$ to $\frac{1}{3}\bar{X}_2 + \frac{1}{3}\bar{X}_3 + \frac{1}{3}\bar{X}_4$

The restricted models for the first three are identical to the same hypothesis in the previous section on Duncan's multiple range test, and the first three columns of Table 2 are relevant. For the final hypothesis, the restriction is

$$b_1 = \frac{1}{3}b_2 + \frac{1}{3}b_3 + \frac{1}{3}b_4 = \frac{1}{3}(b_2 + b_3 + b_4) = b_{14}$$

Since the full model is

$$Y = b_0U + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + E_1,$$

the restricted model is

$$Y = b_{13}U + b_{14}X_1 + \frac{1}{3}b_{14}(X_2 + X_3 + X_4) + E_4$$

$$Y = b_{13}U + \frac{b_{14}}{3}(3X_1 + X_2 + X_3 + X_4) + E_4$$

The full model, of course, is the same as equation (1). For the restricted model, R = .56153, with R^2 = .31532. Using equation (4), $F' =$ 22.3441. As was the case for Duncan's multiple range test in Table 2, a table can be made for Dunn's "c" test. Before constructing the table, the t value is found by the transformation $t = F'$. These values for the first three comparisons are the same as in Table 2. For the last comparison, $t = \sqrt{22.3441} = 4.832$ Table 3 contains the comparisons listed in this section, using Dunn's "c" test as the multiple comparison technique.

Table 3

Dunn's "c" Test In A Regression Formulation

Comparison	F'	$\sqrt{F'} = t$	Region of Rejection at .05 level	Decision
$\bar{X}_1$ to $\bar{X}_2$	.5414	.735	$t \geq 2.818$	Retain
$\bar{X}_3$ to $\bar{X}_4$	.5414	.735	$t \geq 2.818$	Retain
$\bar{X}_1$ to $\bar{X}_3$	19.4602	4.411	$t \geq 2.818$	Reject
$\bar{X}_1$ to $\frac{1}{3}\bar{X}_2 + \frac{1}{3}\bar{X}_3 + \frac{1}{3}\bar{X}_4$	22.3441	4.832	$t \geq 2.818$	Reject

The critical values for this test are obtained from tables in Dunn's article. Again, these values are used rather than using the F distribution or the t distribution directly; the reason for using these tables is to preserve the apparent probability level.

Scheffé's Test

Scheffé's test will allow <u>any</u> comparison to be made, including any <u>a posteriori</u> comparisons that might be interesting to the researcher. This test does, however, have an accompanying loss of power. The same procedure for definition of full and restricted models is used (as was the case in the two previous sections of Duncan's multiple range test and Dunn's "c" test). The difference lies in the distribution to which the value found from equation (4) is to be compared; the correct distribution to be compared to is $(k-1)_{\alpha} F_{k-1,\, N-k}$.

While it is impossible to list all comparisons that might be considered (there are an infinite number of such comparisons), it should be pointed out that beyond the seven comparisons given in the two previous sections, comparisons such as:

$$\tfrac{1}{9}\bar{X}_1 + \tfrac{8}{9}\bar{X}_2 = \tfrac{3}{7}\bar{X}_3 + \tfrac{4}{7}\bar{X}_4$$

can be considered. The restrictions on the regression coefficients for such a comparison would be:

$$\tfrac{1}{9}b_1 + \tfrac{8}{9}b_2 = \tfrac{3}{7}b_3 + \tfrac{4}{7}b_4.$$

A simpler expression of these restrictions is:

$$b_1 + 8b_2 = 3b_3 + 4b_4.$$

The same comparisons listed earlier are considered from the point of view of Scheffé's test, and the results can be found in Table 4.

Table 4

Scheffé's Test In a Regression Formulation

Comparison	F'	Region of Rejection at .05 level	Decision
$\bar{X}_1$ to $\bar{X}_2$	.5414	$F' \geq 9.72$	Retain
$\bar{X}_1$ to $\bar{X}_3$	19.4602	$F' \geq 9.72$	Reject
$\bar{X}_1$ to $\bar{X}_4$	26.4875	$F' \geq 9.72$	Reject
$\bar{X}_2$ to $\bar{X}_3$	13.5144	$F' \geq 9.72$	Reject
$\bar{X}_2$ to $\bar{X}_4$	19.4602	$F' \geq 9.72$	Reject
$\bar{X}_3$ to $\bar{X}_4$	.5414	$F' \geq 9.72$	Retain
$\bar{X}_1$ to $\frac{1}{3}\bar{X}_2 + \frac{1}{3}\bar{X}_3 + \frac{1}{3}\bar{X}_4$	22.3441	$F' \geq 9.72$	Reject

The region of rejection is defined by $(k-1)_{\alpha} F_{k-1,\ N-k}$ which is $3(3.24) = 9.72$.

SUMMARY

In using multiple regression as a problem solving technique, one problem that might arise is the overuse of a full model with several restricted models, without adjusting the probability level. Such an approach would violate the apparent probability level. This has long been a concern in statistics. Several multiple comparison procedures have been developed for different situations.

The intent of the present paper has been to extend some of the better known multiple comparison procedures to a multiple regression approach. The major change in the regression approach is to assess the result of multiple uses of a full model to a correct distribution, rather than a straight-forward usage of the F distribution.

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